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Abstract. Streaming networks have a wide range of options for practical application: information flows in information and communications networks, transport traffic, electrical power systems. The tasks of large dimension, including those which are associated with finding the frequency-optimum traffic, should be solved by the decomposition method. The paper considers the algorithm of the "pinwheel" decomposition of the installation and switching space and synthesis of the streaming network with optimal adjustment placement of different-sized radio elements on the surface of the construct.

1. Introduction

It is difficult to imagine the functioning of modern society without a complex of networks that include such diverse objects as telephone networks, information and telecommunications, pipelines for the delivery of gas and oil, airlines, electrical networks. Networks are systems in which the interaction of elements is carried out, a particular flow is transmitted over the network.

For the purpose of finding solutions to all these tasks, it is convenient to use flows on graphs and in networks, in particular, the tools of the theory of pre-fractal graphs are very effective.

Pre-fractal graphs [1, 3] can be used for the description and modeling of debris flows, river streams, avalanche processes, and transport traffic [3] on the road system, delivery of goods by railways, oil and gas piping distribution systems from the source to the destination.

Pre-fractal graphs proved to be particularly convenient when modeling the optimal decomposition of the installation and switching site (ISS). Each passive (transit) "plate" of the ISS with an increase in the number of radio elements per unit is replaced by a seed, which is the proposed 1-envelope graph. The process of "cutting" or decomposition of the original ISS becomes automatic.

2. Research methods and materials

Here is a complete and strict definition of the installation and switching space (ISS) – the installation and switching space of an i-th rank block is a certain area limited by the dimensions of this block. The installation and switching space is a metric space in which blocks (of i-1th rank) are placed and their electrical connection is carried out. The method of dividing the installation and switching space into parts is the basis of spatial decomposition methods.

The initial data for further work with the ISS and its optimal cutting are: the dimensions of the ISS, shape, size and initial placement of elements and forbidden zones, an array with the coordinates of already

placed elements. After sectioning, the ISS becomes a set of conjugated spatial fragments ("zones" or "plates"). The whole set of zones will be divided into zones that do not have elements inside themselves and, therefore, sources and drains of conductors – "passive" or transit zones, and zones containing elements occupying the zone entirely or partially, with all their size or part – "active" zones with sources and drains, these are the places where conductors "are born" and "die".

To raise the generality, we will consider only the irregular ISS, in which it is impossible to determine the coordinates of the positions in advance, there are no precisely defined seats, there is no constant step between the elements, the placed components have different sizes and shapes.

Initially, we can describe a list of stages in solving a class of design problems associated with the optimal placement of elements in the ISS and the creation of conditions for subsequent successful routing:
- coding of the schematic circuit diagram;
- placement in accordance with the requirements of the subsequent routing (the so-called joint solution of the placement and routing problems);
- topological routing;
- structural design.

The main difference of this approach from all known ways of joint solution of the placement and routing problems is that it operates not with vertical and horizontal highways, channels and local constraints on them. In the new approach, the main highway (channel) degenerates into several parts of the plane ("plate"), in which the desired number of links of the shortest length is guaranteed by their bandwidth capacity in the three coordinates (x-axis, y-axis, the number of the printed layer Z). In the installation space this guarantee is provided by the device called "flows in networks".

The next step is to combine the fragments and highlight the characteristics of the extended object. A necessary condition for the laying of circuits in the form of conductors is the presence of a free resource of the installation and switching space in each arbitrary section so that the bandwidth of any section on the plane was not less than the number of links laid. At the same time, it is necessary to minimize the length of the laid links.

The main tool in solving design problems in the installation and switching spaces of a large size is their spatial decomposition.

The principle of spatial decomposition in the installation and switching space externally seems to be quite simple. It includes: division of space (the whole construct) into spatial fragments, organization of fragments into a hierarchical structure to facilitate the memorization, retrieval and recovery of information from separate fragments, controlled connection of information about objects that occupy several fragments, and the restoration of these objects.

We distinguish three groups (or phases) of problems in the general concept of constructing an optimal model of the ISS by spatial decomposition:
- methods of space division, optimal division of a spatially extended object into spatial fragments;
- solving local problems within spatial fragments (sequentially from one fragment to another, parallelly with all fragments, sequentially with iterations, etc.) with obtaining particular results;
- solution of the communication problem, "cross-linking", aggregation of the obtained partial (local) solutions from separate spatial fragments into a united (general, global) description.

Each of these tasks is an optimization task. For example, placement and routing tasks are interrelated because the quality of the final placement is evaluated by indirect criteria determined by the routing.

3. Findings

We propose a method of recursive division of the installation and switching space with the creation of an active-passive graph model.

The basic principles of the ISS sectioning are as follows:

1. Space is divided into fragments, forming a kind of mosaic, covering the entire area, limited by the dimensions of the i-th block rank without gaps. The plane of each layer is divided by a system of subsets of mutually intersecting lines of variable pitch into separate spatial fragments ("plates"). We can precisely determine the blocks of (i-1)-th rank in the ISS and the zones that do not go beyond the dimensions of the

i-th rank block.

2. The set of mosaic fragments is divided into two mutually disjoint subsets – "active" zones and "passive" ("transit") zones of the ISS. We define active zones as zones that emit or absorb electrical conductors, and passive zones are those zones which are transit, while passing through, conductors are neither absorbed nor created.

3. Often the sides of active and passive zones (blocks of (i-1)-th rank) are parallel to the sides of the block of i-th rank. Usually the boundaries of the i-th rank are orthogonal, in this case the blocks of (i-1)-th rank are represented by rectangles.

4. Each boundary of the active zone is adjacent to the boundaries of one or more passive zones. An unambiguous correspondence with a minimum number of passive zones exists when the boundary of the active zone is adjacent to the boundary of one passive zone.

5. In order to unify sectioning using the principle of its recursiveness (see paragraph 6), the number of fragments must be a multiple of the number of active zones.

6. The principle of sectioning recursiveness, used for its effective and economical execution, is most conveniently implemented and quickly calculated on the computer and implies a sequential enumeration of 1, 2, ..., j, ..., NA active zones, finding the position of the j-th active zone and adjacent fragments (with j =1 occupying the entire dimension of the block of the i-th rank), further allocation of adjacent fragments in some k-th fragment (j+1)-th of the active zone, now occupying the dimension of the k-th fragment only, etc. recursively.

An important role in the installation and switching decomposition belongs to the "walls" between the spatial fragments, as well as to the location of transit points that allow connections (conductors) to go through the walls. There is a possibility of more flexible resource allocation of the "walls", the flows of connections with the bandwidth capacity of the "walls" are determined, considering that the conductors are not fixed on the "walls", but the concept of "beams", "bundles", "flows" of connections and the mathematical apparatus of flows in networks, graph or prefractal graph is used. Flows in networks make it possible to flexibly direct the flow of printed conductors through the space, it requires consideration of sources, intermediate vertices and drains. On the metric model of the installation and switching space, this leads to the appearance of "active" zones (generating or absorbing conductors) and "passive" ("transit") zones (with conductors only passing through). To divide the space, special techniques of its sectioning, optimal zone construction, introduction of dual graphs showing the completed transformation are required.

Let us consider a simple example with the solution of all problems simultaneously. We take two sources (drains) of links beginning (ending) in this source (drain are initially placed on a flat single-layer construct. We suppose that they are called i and j and let it always be so that i < j. As we will see further, the order of the active zone sectioning process is essential and will require a special apparatus for solving the multicriteria problem of placing centers on multi-weighted prefractal graphs.

If the above-described sectioning of the construct is carried out (from the smallest number of the i source or drain), these transformations are converted into a dual graph (network), shown in figure 1(a) then you can see some interesting properties of the dual graph:

1. All edges of the graph will have very simply calculated "wall" bandwidth capacity, for example:

$C23i = LXi – Xi - Ai$;

$C02i = Ai$;

$C34i = Yi …$

where LXi is the size of the construct on the horizontal x-axis;

Xi - coordinates on the x-axis of the lower left corner of the element;

Yi - y coordinate of the lower left corner of the element;

Ai - the element size in the x-axis.

2. The number of such zones and connecting edges increases with the number of "active elements" of the scheme N as 4N + 2 and 12N;

3. The movement of the "active elements" on the surface of the construct by $\pm \Delta X$ and $\pm \Delta Y$ is easily and unambiguously reflected in the bandwidth of the edges of the dual graph. We suppose that the i zone

will move $+ \Delta X_i$ to the right and $+ \Delta Y_i$ up (figure 1), then:

$C*_{23i} = LX_i – X_i - A_i - \Delta X_i = C_{23i} - \Delta X_i$;

$C*_{02i} = A_i = C_{02i}$;

$C*_{34i} = Y_i + \Delta Y_i = C_{34i} + \Delta Y_i$;…

4. The variation in the size of the "active" and forbidden zones (let zones i be $+\Delta A_i$ and $+\Delta B_i$) is unambiguously reflected in the change in the bandwidth of some edges of the graph:

$C**_{23i} = LX_i – X_i – A_i - \Delta A_i = C_{23i} - \Delta A_i$;

$C**_{02i} = A_i + \Delta A_i = C_{02i} + \Delta A_i$;

$C**_{34i} = Y_i = C_{34i}$;…

The eventually created dual graph of such a decomposition will be called an "envelope graph", it has many interesting analytical properties, in particular, it is planar and three times colorable. Figure 3 shows 1-envelope-graph (a) and 2-envelope-graph (b).

4. Discussion

Let us consider how the supply and demand problem is solved in the installation and switching space. Each "source" or "drain" of electrical connections (for example, a chip) with its supply in $A(x_i)$ units or demand in $B(y_j)$ units creates a flow that must satisfy the demand by supply, do not exceed the bandwidth capacity of the edges (sections), minimize the flow cost if by the cost $d(x_k, y_l)$ of delivering of a product unit from x_k to y_l we understand the length of the connection (distance) between the elements x_k and y_l.

For an arbitrary graph (network) $L(X,U;P)$ having S sources, intermediate R vertices-transiters ("passive" zones and corresponding vertices in the envelope graph) and drains T, $f(x, y)$ – the flow on the edge (x,y), bandwidth capacity of edges $c(x, y)$, edge lengths $d(x, y)$, supply $A(x)$ at vertices x S and demand $B(x)$ at vertices $x \in T$, the problem is to construct a permissible flow, minimizing the length of the links, i.e.

$f(x, X) – f(X, x) \leq A(x)$ $x \in S$

$f(x, X) – f(X, x) = 0$ $x \in R$

$f(x, X) – f(X, x) \leq B(x)$ $x \in T$

$0 \leq f(x,y) \leq c(x,y)$ $(x,y) \in U$

minimize $\sum to(x,y)$ $f(x,y)$,

where $A(x)$, $B(y)$, $C(x,y)$ are positive;

$d(x,y)$ are non-negative integers.

The solution of the problem gives the necessary conditions for the subsequent routing – in any section of the installation and switching space the resource is sufficient for the shortest length of the routes.

The next step in using the principles of spatial decomposition is the synthesis of such a graph (network) in which the flow exists and satisfies all externally imposed constraints. In the description of the envelope-graph, the variation of the bandwidth capacity of its edges is directly proportional to the variations in the position of the "active" zone (of a radio element) in the installation and switching space or to the variations in the size of these zones. By moving the zone, changing its size, we increase the bandwidth of the "narrow" section (of the corresponding edge of the graph) by reducing the bandwidth capacity of excess free space. For example, figure 1, a shows that the right section is overloaded with the flow of conductors, and the left one is underloaded. If on the right the bandwidth C_{23} was not enough to place the conductors, then by moving the i chips to the left we will open the section or at least level up the bandwidth capacity of sections C_{14} and C_{23}. The "dance" of the active zones leads to an increase in the flow in narrow saturated sections at both coordinates at the same time, so that the "maximum flow" satisfying the demand with supply through the network begins to be built.

The problem of network synthesis is posed more strictly in the paradigm of the "flows in networks" methodology, in which a symmetric function is given so that the flow on all edges of the graph is:

$f(x, y) \leq r(x < y)$ $x, y \in X$.

In this case, a permissible network that minimizes some given edges' bandwidth capacity is constructed, for example:

$\sum d(x, y)c(x, y)$,

where d(x, y) can be understood as the known cost of space between x and y units of edge bandwidth capacity, in our problems this is most often the "length" of the edge in the Eulerian or Manhattan metrics.
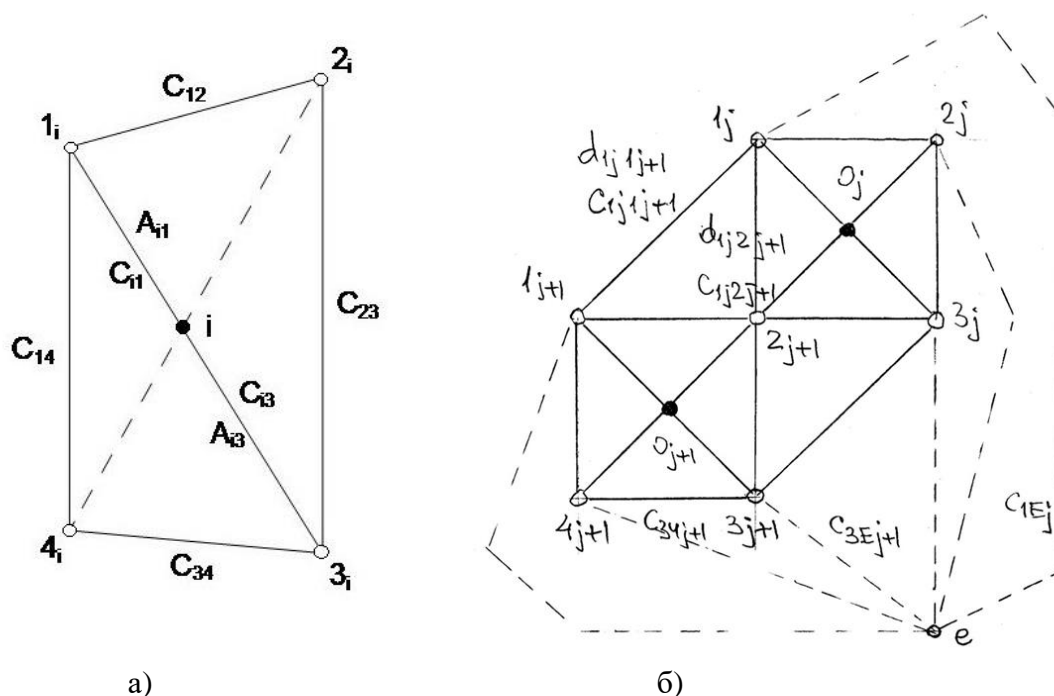

Figure 1. a) 1-envelope-graph as a topological model of a radio element and its surrounding zones of the sectioned installation and switching space;
b) 2-envelope-graph as a topological model of sequential placement of two radio elements in the installation and switching space with the introduction of an infinite face E and additional edges of its connection with the original model.

It is easy to see that the specified task does not simply combine the well-known problems of installation and routing but goes much further. It synthesizes such a placement, which in each part of the installation and switching space guarantees successful subsequent routing in topological sense.

In addition, after the separation of the ISS, it is necessary to create some dual-weighted graph, to determine the set of its intermediate vertices and edges, the bandwidth capacity of which would reflect the metric correspondence of the boundaries of the spatial fragments (zones) relative to each other. In particular, the bandwidth capacity of the edges must correspond to:

- the zone metric sizes;
- the sizes of the common parts of the boundaries between zones;
- the bandwidth capacity of the zone boundaries relative to each other.

Similarly, metric characteristics such as length, distance between centers of zones, distance between elements, etc. should be represented and modeled in a weighted graph.

The metric decomposition of the ISS is represented by a dual graph LD(X,U;P), whose vertices xi, xj, xk ϵ X correspond to the geometric centers of the fragments of the partition, and the edges ul = <xi, xj>, ϵ U, note the commonality of the boundaries of the i-th and j-th zones. Many scalar characteristics can be attributed to the edges and vertices of an LD graph, transforming the graph into a network. In particular, we will be interested in such characteristics as:

cij – the upper limit of the bandwidth capacity of the common boundary of the i-th and j-th zones, the length of this boundary in some standard discrete units;

dij – distance between conditional, geometric centers of i and j zones;

Ψk – the upper limit of the bandwidth capacity of the vertex xk, i.e. the width of some section of the

xk active zone under the element, etc.

We introduce an infinitely large "outer" E zone and, correspondingly, the e vertex and the edges connecting e to the centers of the boundary zones. In figure 1, b they will be denoted by a dotted line. Obtained multi-pole graph will be called "wheel-graph". Since the LD graph is planar and its faces are clearly depicted, the Euler formula holds for it (for planar graphs):

Fp(LD) + N(LD) – M(LD) = 2

The "pinwheel" method of the ISS division into zones supposes that rays are emitted from the corners of each rectangular active zone until they intersect with the boundaries of the construct, other active elements or rays already drawn from them.

The edges and vertices of the envelope-graph can be provided with the following scalar characteristics: the upper limit of the "walls" bandwidth capacity with (xi, xj), the distance between the geometric centers of the plates d(xi, xj), etc. NB, the distances can be measured in different metrics: Euclidean, Manhattan ("modulo"), etc. Due to the rectangular specificity of the ISS, the Manhattan metric is used in this paper.

The theory of algorithms considers a problem solvable if there is an algorithm that solves it. However, an important part of the solution is the implementation of the algorithm. It turns out that some algorithms require such a large number of elementary steps that the solution of the problem is impossible in the foreseeable future.

In theory, the judgmental, quantitative by nature concept of solvability is introduced, serving as a criterion for the possibility and feasibility of its practical application. The most important characteristics of these algorithms are the time and memory consumed during the calculation. However, it is known that time reflects the complexity of the algorithm more subtly than memory. Therefore, we will further mention the time complexity of algorithms and problems solved by algorithms.

Typically, some positive integer n connected with the scope of the problem is taken as a parameter that characterizes the initial data of the algorithm. Then the algorithm efficiency with respect to its operation time is compared with the growth rate of the time estimate as a function of F(n).

In the analyzed reviews, time estimates F(n) of the "flows in networks" algorithm are given in 7 sources: $O(|X| \cdot |U|^2)$; $O(|X|^2 \cdot |U|)$; $O(|X|^3)$; $O(|X|^2 \cdot |U|^{1/2})$; $O(|X|^{5/3} \cdot |U|^{2/3})$; $O(|X| \cdot |U| \cdot log^2 |X|)$; $O(|X| \cdot |U| \cdot log |X|)$. Thus, for the case of a spatial decomposition partition with a stream connection of fragments, the solution time estimate varies from $O(Q2.33)$ to $O(Q3)$, while for the usual decomposition it varies from $O(Q3)$ to $O(Q5)$.

The "cross-linking" of particular solutions into a unified final solution is the ultimate stage in the spatial decomposition method. Obvious requirements to the components of this result are imposed:

- obtaining the results of particular (local) solutions at the input with the unified (global) solution making at the output;
- the power-law dependence of the problem solution time F(n) on the scope of the problem takes place, in mathematical terms, if there is a reducibility of the search problem P1 to the search problem P2, which is such a transformation that can be carried out in polynomial time. The degree of such a polynomial should not be large, no more than 3-4;
- mathematical transformations at the "cross-linking" stage should be most fully consistent with the essence of physical transformations in the source system for direct, visual and rapid processing of the ultimate results;
- the tasks of situation analysis should be supplemented by the tasks of synthesis of the optimal result.

5. Conclusion

Thus, the principle of spatial decomposition is proposed in this paper, the model of installation and switching space (ISS) is constructed, the optimal ISS is synthesized from the standpoint for its better traceability.

The optimal method of spatial recurrent decomposition of the installation and switching space with the division of vertices into active (where the flows are generated or absorbed) and passive (where the flows pass through), with the minimization of the number of passive spatial fragments, which provides

pragmatic convenience, saves computer time, is proposed. Eventually, the optimal spatial decomposition reveals cuttings and sections, determines the bandwidth capacity of the sections of the ISS;

In order to build a streaming model, a mathematical apparatus of "flows in networks" was chosen, which allows to solve the problem of maximum flow, the problem of demand and supply at minimum cost, a multipolar problem of maximum flow in a network with many sources and drains, the problem of synthesis of a network with a given bandwidth capacity of the sections.

In the paradigm of the "flows in networks" methodology, the problem of the network synthesis is posed in the ISS, it specifies the symmetric function to the flow on all edges of the graph $f(x, y) \leq r$ ($x < y$) x, y, X. The permissible network is synthesized, minimizing the given function of the bandwidth capacity of edges, for example $\sum d(x, y)c(x, y)$, where $d(x, y)$ is understood as the known cost of the space between x and y units of the edge bandwidth capacity, in the problems of measures, it is the "length" of the edge in Eulerian or Manhattan metrics.

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